Orange Public Schools

Office of Curriculum & Instruction

2019-2020 Mathematics Curriculum Guide

Fourth Grade

Unit 7:

Eureka - Module 7: Exploring Measurement with Multiplication

June 1, 2020 – End of School Year

Board Approved: 1.14.2020

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Yearlong Pacing Guide: Second Grade

Eureka Math	*Eureka Module Standards*
Unit 1- Module 1: Place Value, Round, and Algorithms for Addition/ Subtraction **Sept 9- Oct 18**	4OA3, 4NBT1, 4NBT2, 4NBT3, 4NBT4
Unit 2- Module 3: Multi-Digit Multiplication and Division **Oct 21- Dec 20**	4OA1, 4OA2, 4OA3, 4OA4, 4NBT5, 4NBT6, 4MD3
Unit 3- Module 5: Fractions Equivalence, Ordering, and Operations **Jan 2- March 6**	4NF1,4NF2, 4NF3, 4NF4, 4MD4
Unit 4- Module 6: Decimal Fractions **March 9- April 9**	4NF5,4NF6, 4NF7,4MD2
Unit 5- Module 4: Angle Measures and Plane Figures **April 20- May 15**	4MD5, 4MD6, 4MD7, 4G1, 4G2, 4G3
Unit 6- Module 2: Unit Conversions and Problem Solving **May 18- May 29**	4.MD.1, 4.MD.2
Unit 7- Module 7: Exploring Multiplication **June 1- EOSY**	4OA1, 40A2, 40A3, 4MD1, 4.NBT5, 4.NBT6 4MD2

References

"Eureka Math" *Gt Minds.* 2018 < https://greatminds.org/account/produc

Table of Contents

Module 7

Essential Questions

- How are the four basic operations related to one another?
- How are the units of measure within the metric system related?
- How do you find the area and perimeter of geometric figures and how can using the formulas for perimeter and area help you solve real-world problems?
- What real-life situations require the use of multiplication or division?

Enduring Understandings

- The four operations are interconnected.
- Converting from larger to smaller units of measurement in the metric system is done by multiplying by powers of ten.
- Perimeter is a real-life application of addition and subtraction.
- Area is a real-life application of multiplication and division.
- There are three different structures for multiplication and division problems: Area/Arrays, Equal Groups, and Comparison, and the unknown quantity in multiplication and division situations is represented in three ways: Unknown Product, Group Size Unknown, and Number of Groups Unknown
- Represent verbal statements of multiplicative comparisons as multiplication equations.

Performance Overview

- In this module, students build their competencies in measurement as they relate multiplication to the conversion of measurement units. Throughout the module, students explore multiple strategies for solving measurement problems involving unit conversion.
- In Topic A, students build on their work in Module 2 with measurement conversions. Working heavily in customary units, students use two-column conversion tables (4.MD.1) to practice conversion rates. For example, following a discovery activity where students learn that 16 ounces make 1 pound, students generate a two-column conversion table listing the number of ounces in 1 to 10 pounds. Tables for other measurement units are then generated in a similar fashion. Students then reason about why they do not need to complete the tables beyond 10 of the larger units.
- Topic B builds upon the conversion work from Topic A to add and subtract mixed units of capacity, length, weight, and time. Working with metric and customary units, students add like units, making comparisons to adding like fractional units, further establishing the importance of deeply understanding the unit.
- In Topic C, students reason how to convert larger units of measurements with fractional parts into smaller units by using hands-on measurements. For example, students convert 3 1 4 feet to inches by first finding the number of inches in 1 4 foot. They partition a length of 1 foot into 4 equal parts and find that 1 4 foot = 3 inches. They then convert 3 feet to 36 inches and add 3 inches to find that 3 1 4 feet = 39 inches.

Module 7: Exploring Measurement with Multiplication

Pacing: June 1- End of School Year Suggested Instructional Days: 24		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Measurement Conversion Tables	Lesson 1	Create conversion tables for length, weight, and capacity units using measurement tools, and use the tables to solve problems. https://www.youtube.com/watch?v
	Lesson 2	Create conversion tables for length, weight, and capacity units using measurement tools, and use the tables to solve problems. https://www.youtube.com/watch?v
	Lesson 3	Create conversion tables for units of time, and use the tables to solve problems. https://www.youtube.com/watch?v
	Lesson 4	Solve multiplicative comparison word problems using measurement conversion tables. https://www.youtube.com/watch?v
	Lesson 5	Share and critique peer strategies. https://www.youtube.com/watch?v
Topic B: Problem Solving with Measurement	Lesson 6	Solve problems involving mixed units of capacity. https://www.youtube.com/watch?v
	Lesson 7	Solve problems involving mixed units of length. https://www.youtube.com/watch?v
	Lesson 8	Solve problems involving mixed units of weight. https://www.youtube.com/watch?v
	Lesson 9	Solve problems involving mixed units of time. https://www.youtube.com/watch?v
	Lesson 10	Solve multi-step measurement word problems. https://www.youtube.com/watch?v
	Lesson 11	Solve multi-step measurement word problems. https://www.youtube.com/watch?v

Topic C: Investigation of Measurements Expressed as Mixed Numbers	Lesson 12	Use measurement tools to convert mixed number measurements to smaller units. https://www.youtube.com/watch?v
	Lesson 13	Use measurement tools to convert mixed number measurements to smaller units. https://www.youtube.com/watch?v
	Lesson 14	Solve multi-step word problems involving converting mixed number measurements to a single unit. https://www.youtube.com/watch?v
End- Of- Module Assessment		
Topic D: Year in Review	Lesson 15	Create and determine the area of composite figures.
	Lesson 16	Create and determine the area of composite figures.
	Lesson 17	Practice and solidify Grade 4 fluency.
	Lesson 18	Practice and solidify Grade 4 vocabulary.

Modifications	
Special Education/ 504:	**English Language Learners:**
-Adhere to all modifications and health concerns stated in each IEP. -Give students a menu of options, allowing students to pick assignments from different levels based on difficulty. -Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time -Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or in writing, such as Read-Draw-Write -Provide breaks between tasks, use positive reinforcement, use proximity -Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18) - Strategies for Students with 504 Plans	- Use manipulatives to promote conceptual understanding and enhance vocabulary usage - Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction - During i-Ready lessons, click on "Español" to hear specific words in Spanish - Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information - Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems - Utilize program translations (if available) for L1/ L2 students - Reword questions in simpler language - Make use of the ELL Mathematical Language Routines (click here for additional information) -Scaffolding instruction for ELL Learners -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)
Gifted and Talented:	**Students at Risk for Failure:**
- Elevated contextual complexity - Inquiry based or open ended assignments and projects - More time to study concepts with greater depth - Promote the synthesis of concepts and making real world connections - Provide students with enrichment practice that are imbedded in the curriculum such as: • Application / Conceptual Development • Are you ready for more? - Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20) - Provide opportunities for math competitions - Alternative instruction pathways available	- Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum - Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Field Trips, Google Expeditions, Peer Support, one on one instruction - Assure constant parental/ guardian contact throughout the year with successes/ challenges - Provide academic contracts to students/ guardians - Create an interactive notebook with samples, key vocabulary words, student goals/ objectives. - Always plan to address students at risk in your learning tasks, instructions, and directions. Try to anticipate where the needs will be and then address them prior to lessons. -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)

21st Century Life and Career Skills:

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

https://www.state.nj.us/education/cccs/2014/career/9.pdf

- **CRP1**. Act as a responsible and contributing citizen and employee.
- **CRP2**. Apply appropriate academic and technical skills.
- **CRP3**. Attend to personal health and financial well-being.
- **CRP4**. Communicate clearly and effectively and with reason.
- **CRP5**. Consider the environmental, social and economic impacts of decisions.
- **CRP6**. Demonstrate creativity and innovation.
- **CRP7**. Employ valid and reliable research strategies.
- **CRP8**. Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP9**. Model integrity, ethical leadership and effective management.
- **CRP10**. Plan education and career paths aligned to personal goals.
- **CRP11**. Use technology to enhance productivity.
- **CRP12**. Work productively in teams while using cultural global competence.

Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.

Technology Standards:

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.

https://www.state.nj.us/education/cccs/2014/tech/

8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
C. **Communication and Collaboration:** Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
D. **Digital Citizenship:** Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
F. **Critical thinking, problem solving, and decision making:** Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

A. **The Nature of Technology: Creativity and Innovation-** Technology systems impact every aspect of the world in which we live.
B. **Technology and Society:** Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
C. **Design:** The design process is a systematic approach to solving problems.
D. **Abilities in a Technological World:** The designed world in a product of a design process that provides the means to convert resources into products and systems.
E. **Computational Thinking: Programming-** Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

Interdisciplinary Connections:	
English Language Arts:	
RF.4.4	Read with sufficient accuracy and fluency to support comprehension.
W.4.10	Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.
SL.4.1	Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 4 topics and texts*, building on others' ideas and expressing their own clearly.

NJSLS Unpacked Standards	
4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., “a is n times as much as b”).

- Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

Example:
5 x 8 = 40. Sally is five years old. Her mom is eight times older. How old is Sally’s Mom? 5 x 5 = 25 Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

- Utilize the properties and patterns of multiplication (including the commutative, associative, and identity properties). Identify and verbalize which quantity is being multiplied and which number tells by how many times. Explore the meaning of the two factors in comparison multiplication problems. Practice writing and identifying equations and statements for multiplicative comparisons. Use manipulatives to represent how many times greater the area of one shape is than another, such as pattern blocks. Provide contextualized situations which make use of diagrams, a table, and equations.

Example: There were thirty-two adults and four children in line at a movie theater. How many times more adults were in the line than children?

- Utilize multiplicative thinking, known multiples, and the meaning of each factor/product. Interpret diagrams that focus on unmeasured multiplicative relationships. Explain multiplication equations to represent comparisons. Promote words like “doubling” and “tripling” to connect to “two times as much” and “three times as much” to introduce multiplicative relationships. Distinguish multiplicative comparison from the additive comparison.

- A situation that can be represented by multiplication has an element that represents the scalar and an element that represents the quantity to which the scalar applies. (NCTM, Essential Understanding, 2011).

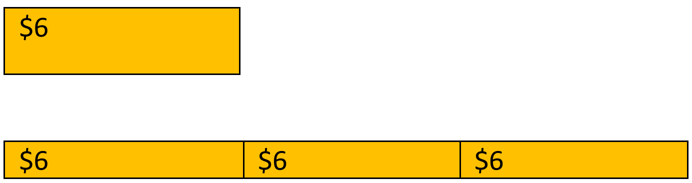
- A multiplicative comparison involves a constant increase that x is more times or x times less; whereas an additive comparison only involves determining how many more than or how many less than another set.

- One of the factors in multiplication indicates the number of objects in a group and the other factor indicates the number of groups. In the multiplicative expression A x B, A can be defined as a scaling factor. (NCTM, Essential Understanding, 2011).

4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

- This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.

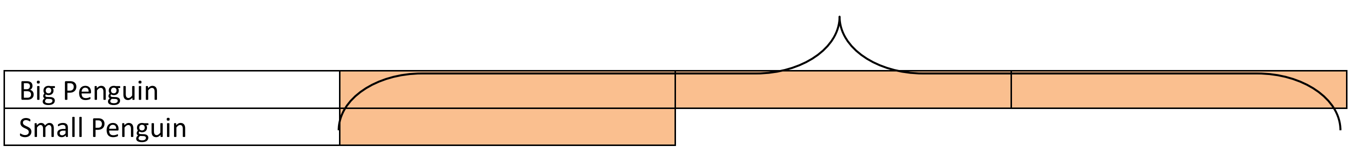
- In an additive comparison, the underlying question is *what amount would be added to one quantity* in order to result in the other. In a multiplicative comparison, the underlying question is *what factor would multiply one quantity* in order to result in the other.



3 x *B*=*R*
3 x $6 = $18

A tape diagram used to solve a Compare problem:

A big penguin will eat 3 times as much fish as a small penguin. The big penguin will eat 420 grams of fish. All together, how much will the two penguins eat?



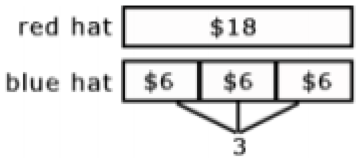
B=number of grams the big penguin eats
S=number of grams the small penguin eats
3 x S=B
3 x S=420
S=140
S + B=140 + 420 =560

- Identify when multiplication or division must be used in a multiplicative comparison question. For example, when "5 times more" indicates multiplication and "5 times less" indicates division. Identify what amount would be added to or subtracted from one quantity in order to result in the other. Recognize the inverse relationship between multiplication and division, and determine that division can be used to solve comparison multiplication problems when either group size or the scaling factor is provided. Tape diagrams can be used as a strategy in solving problems with multiplicative comparisons.

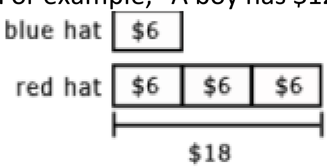
<u>*There are three kinds of multiplicative comparison word problems:*</u>

Product unknown comparisons.

For example, "A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?"



Set size unknown comparisons,
For example, "A boy has $12 and each red hat costs $3. How many hats can the boy buy?"



Multiplier unknown comparisons,
For example, "A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?"

- Students should be able to distinguish multiplicative comparison from additive comparisons. Assess students' interpretations of a model in use to determine whether their view demonstrates mathematical understanding. Translate comparative situations into equations. Represent problems with drawings and equations, using a symbol for the unknown number. *Find evidence in a word problem to help develop an equation and support the operation chosen to solve.*

4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?
Some typical estimation strategies for this problem:

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530. The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550).

- Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies.

Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

- The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, millimeter, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students develop benchmarks and mental images about a meter (e.g., about the height of a tall chair) and a kilometer (e.g., the length of 10 football fields including the end zones, or the distance a person might walk in about 12 minutes), and they also understand that "kilo" means a thousand, so 3000 m is equivalent to 3 km.

- Expressing larger measurements in smaller units within the metric system is an opportunity to reinforce notions of place value. There are prefixes for multiples of the basic unit (meter or gram), although only a few (kilo-, centi-, and milli-) are in common use. Tables such as the one below are an opportunity to develop or reinforce place value concepts and skills in measurement activities. Relating units within the metric system is another opportunity to think about place value. For example, students might make a table that shows measurements of the same lengths in centimeters and meters. Relating units within the traditional system provides an opportunity to engage in mathematical practices, especially "look for and make use of structure" and "look for and express regularity in repeated reasoning" For example, students might make a table that shows measurements of the same lengths in feet and inches.

Super- or subordinate unit	Length in terms of basic unit
kilometer	10^3 or 1000 meters
hectometer	10^2 or 100 meters
decameter	10^1 or 10 meters
meter	1 meter
decimeter	10^{-1} or $\frac{1}{10}$ meters
centimeter	10^{-2} or $\frac{1}{100}$ meters
millimeter	10^{-3} or $\frac{1}{1000}$ meters

Centimeter and meter equivalences

cm	m
100	1
200	2
300	3
500	
1000	

Foot and inch equivalences

feet	inches
0	0
1	12
2	24
3	

- Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12. Example: Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	***n*** x 3

- Foundational understandings to help with measure concepts:
- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).

- Understand the relationship between the size of a unit and the number of units needed
- These Standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity).

4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

- This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, and dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

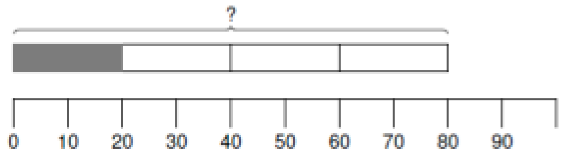
Example:
Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?
Possible solution:
Charlie plus 10 friends = 11 total people
11 people x 8 ounces (glass of milk) = 88 total ounces
1 quart = 2 pints = 4 cups = 32 ounces
Therefore 1 quart = 2 pints = 4 cups = 32 ounces
2 quarts = 4 pints = 8 cups = 64 ounces
3 quarts = 6 pints = 12 cups = 96 ounces
If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.

Additional Examples with various operations:

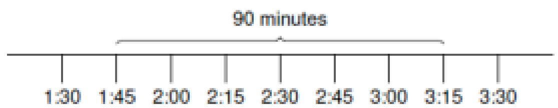
- *Division/fractions:* Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches. Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)
- *Addition:* Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?
- *Subtraction:* A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?
- *Multiplication:* Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?
- Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours

throughout the day, or a volume measure on the side of the container.

Juan spent ¼ of his money on a game. The game cost $20. How much money did he have at first?



What time does Maria have to leave to be at her friend's house by a quarter after 3 if the trip takes 90 minutes?

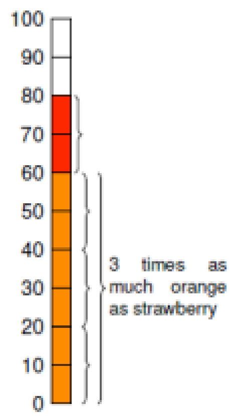


Using a number line diagram to represent time is easier if students think of digital clocks rahter than round clocks. In the latter case, placing the numbers on the number line involves considering movements of the an minute hands.

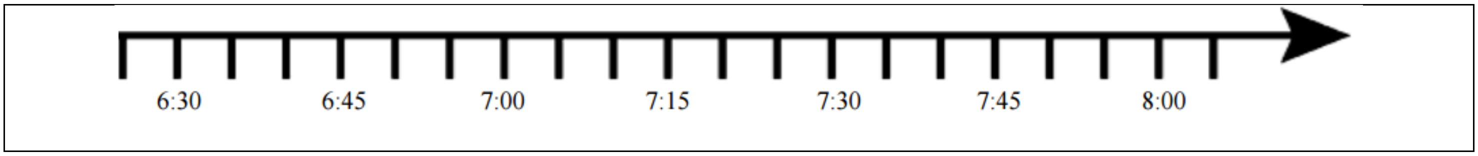
- Students also combine competencies from different domains as they solve measurement problems using all four arithmetic operations, addition, subtraction, multiplication, and division.

Example:
"How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?" Students may use tape or number line diagrams for solving such problems.

Example:
Lisa put two flavors of soda in a glass. There were 80 ml of soda in all. She put three times as much orange drink as strawberry. How many ml of orange did she put in?



Example:
At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.



Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 7 Assessment Framework			
Assessment	**NJSLS**	**Estimated Time**	**Format**
Optional End-of- Module Assessment	4.OA.1-3 4.MD.1-2	1 Block	Individual or Small Group with Teacher
Grade 4 Interim Assessment 4 (i-Ready)	4.MD.5-7 4. G.1-2	1 Block	Individual

Module 7 Performance Assessment/ PBL Framework			
Assessment	**NJSLS**	**Estimated Time**	**Format**
Module 7 Performance Task 1 *Margie's Apples*	4.MD.2	Up to 30 minutes	Individual or Small Group
Extended Constructed Response (ECR)* (click here for access)	Dependent on unit of study & month of administration	Up to 30 Minutes	Individual

Use the following links to access ECR protocol and district assessment scoring documents:

- Assessment and Data in Mathematics Bulletin
- ECR Protocol
-

Fourth Grade Ideal Math Block

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA
Technology Integration: https://embarc.online/
*Website provides Goggle slides, additional activities, and student videos per lesson

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions
Technology Integration: https://embarc.online/
Website provides Goggle slides, additional activities, and student videos. per lesson

Student Debrief: Whole Group
Exit Ticket: Independent

50-60 min.

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

M:
Meet with the teacher
https://teacher-toolbox.com/

A:
Application/ Problem Solving

T:
Technology Resources
I-ready
Zearn

H:
Hands on Activities

20-30 min.

Eureka Lesson Structure:

Fluency:

- Sprints
- Whiteboard Exchange

Technology Integration:

Splat Sequences

Which one doesn't belong?

Would you rather?

Esti- Mysteries

Anchor Task:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Guided Practice/ Independent Practice: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Technology Integration:

Think Central:

- Pre-Test
- Chapter Review
- Test Prep
- Performance Tasks

https://embarc.online/

Virtual Manipulatives for lessons
http://nlvm.usu.edu/en/nav/vlibrary.html
For videos that students can watch and interact with independently click here

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

Centers:

- I-Ready: https://login.i-ready.com/ *i-Ready* makes the promise of differentiated instruction a practical reality for teachers and students. It was designed to get students excited about learning and to support teachers in the challenge of meeting the needs of all learners. Through the power of one intuitive system whose pieces were built from the ground up to work together, teachers have the tools they need to ensure students are on the road to proficiency.
- Zearn: https://www.zearn.org/ **Zearn** Math is a K-5 math curriculum based on Eureka Math with top-rated materials for teacher-led and digital instruction.
- Teacher Toolbox; https://teacher-toolbox.com/ A digital collection of K-8 resources to help you differentiate instruction to students performing on, below, and above grade level.

NJSLA Assessment Evidence/Clarification Statements			
NJSLS	**Evidence Statement**	**Clarification**	**MP**
4.OA.1-1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5	• Tasks have "thin context" or no context.	MP.2,4
4.OA.1-2	Represent verbal statements of multiplicative comparisons as multiplication equations.	• Tasks have "thin context" or no context.	MP 2,4
4.OA.2.	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison	• See the OA Progression document, especially p. 29 and Table 2, Common Multiplication and Division situations on page 89 of CCSSM. • Tasks sample equally the situations in the third row of Table 2 on page 89 of CCSSM	MP 1,4, 5
4.OA.3-1	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations.	• Assessing reasonableness of answer is not assessed here. • Tasks do not involve interpreting remainders.	MP.1,2,7
4.OA.3-2	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, in which remainders must be interpreted	• Assessing reasonableness of answer is not assessed here. • Tasks involve interpreting remainders. • See p. 30 of the OA Progression document. • Multi-step problems must have at least 3 steps.	MP 1,2,4,7

4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12) , (2, 24) , and (3, 36) ,…	• None	MP.5, MP.8
4.MD.2.1	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, in problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	• Situations involve whole-number measurements and require expressing measurements given in a larger unit in terms of a smaller unit. • Tasks may present number line diagrams featuring a measurement scale. • Tasks may include measuring to the nearest cm or mm	MP.4, MP.5
4.MD.2.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, in problems involving simple fractions or decimals. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	• Situations involve two measurements given in the same units, one a whole-number measurement and the other a non-whole number measurement (given as a fraction or a decimal). • Tasks may present number line diagrams featuring a measurement scale. • Tasks may include measuring distances to the nearest cm or mm.	MP.4, MP.5

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

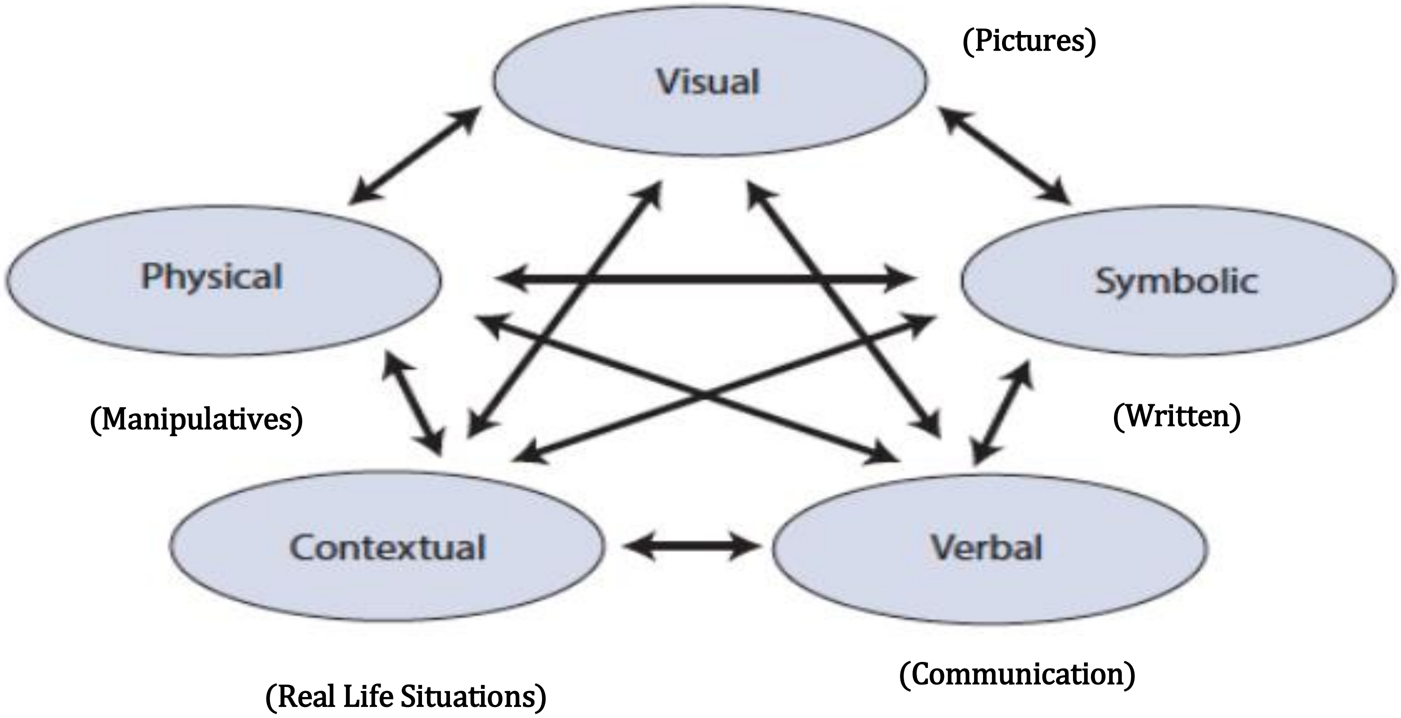
The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?

Student Name: ________________________________Task: ____________________________School: ___________ Teacher: ______________
Date: ___________

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	…a start 1	…getting there 2	…that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

<u>Teacher Questioning:</u>

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48. **What would happen if ___?**
49. Do you see a **pattern**?
50. What are some **possibilities** here?
51. Where could you find the **information** you need?
52. How would you **check your steps** or your answer?
53. What **did not work**?
54. How is your solution method the **same as or different from** [student]'s method?
55. Other than retracing your steps, **how can you determine** if your answers are appropriate?
56. How did you **organize** the information? Do you have a **record**?
57. How could you solve this using **tables, lists, pictures, diagrams**, etc.?
58. What have you tried? What **steps** did you take?
59. How would it look if you used this **model** or these **materials**?
60. How would you draw a **diagram or make a sketch** to solve the problem?
61. Is there **another possible answer**? If so, explain.
62. Is there **another way to solve** the problem?
63. Is there **another model** you could use to solve the problem?
64. Is there anything you've **overlooked**?
65. **How did you think** about the problem?
66. What was your **estimate or prediction**?
67. How **confident** are you in your answer?
68. **What else** would you like to know?
69. What do you think comes **next**?
70. Is the solution **reasonable**, considering the context?
71. Did you have a **system**? Explain it.
72. Did you have a **strategy**? Explain it.
73. Did you have a **design**? Explain it.

Ready

100 Questions That Promote Mathematical Discourse | 3

Help students learn to **connect mathematics, its ideas, and its application**

74. What is the **relationship** between ___ and ___?
75. Have we ever solved a problem **like this before**?
76. What uses of mathematics did you find in the **newspaper** last night?
77. What is the **same**?
78. What is **different**?
79. Did you use skills or build on concepts that were **not necessarily mathematical**?
80. Which **skills or concepts** did you use?
81. What **ideas** have we explored before that were useful in solving this problem?
82. Is there a **pattern**?
83. **Where else** would this strategy be useful?
84. How does this **relate** to ___?
85. Is there a **general rule**?
86. Is there a **real-life situation** where this could be used?
87. How would your method work with **other problems**?
88. What other problem does this seem to **lead to**?

Help students **persevere**

89. Have you tried making a **guess**?
90. **What else** have you tried?
91. Would **another method** work as well or better?
92. Is there **another way** to draw, explain, or say that?
93. Give me another **related problem**. Is there an easier problem?
94. How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95. What was **one thing you learned** (or two, or more)?
96. Did you **notice any patterns**? If so, describe them.
97. What **mathematics topics** were used in this investigation?
98. What were the **mathematical ideas** in this problem?
99. What is mathematically **different about these two situations**?
100. What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation
3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

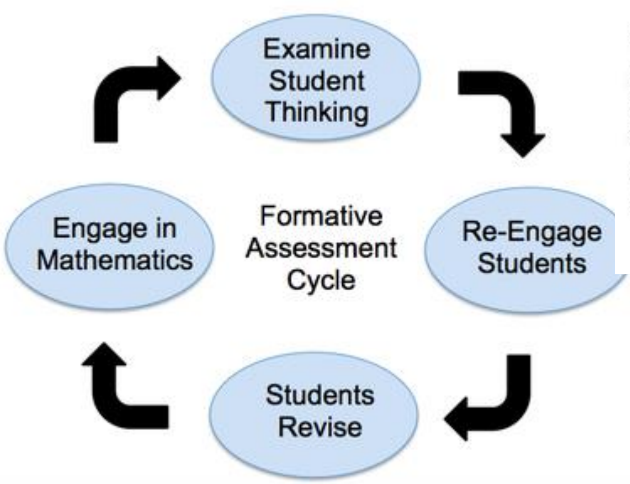
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others** In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
4	**Model with mathematics** Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

5	**Use appropriate tools strategically** Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision** As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure** In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning** Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ **Time:** _______________

NJSLS.:

____

____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE **Week of: ________________**

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	

Fourth Grade PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		**Not There Yet** Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% **Distinguished command**	**PLD Level 4: 89%** **Strong Command**	**PLD Level 3: 79%** **Moderate Command**	**PLD Level 2: 69%** **Partial Command**	**PLD Level 1: 59%** **Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

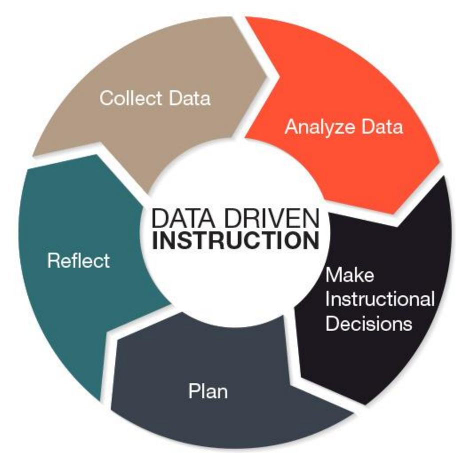
Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?



Data Analysis Form **School**: ______________**Teacher**: __________________________ **Date**: _______________

Assessment: _______________________________________**NJSLS**: _______________________________________

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

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MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the NJSLS. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

4th Grade Authentic Assessment #1 – Margie's Apples

Name:____

Margie bought 3 apples that cost 50 cents each. She paid with a five-dollar bill. How much change did Margie receive?

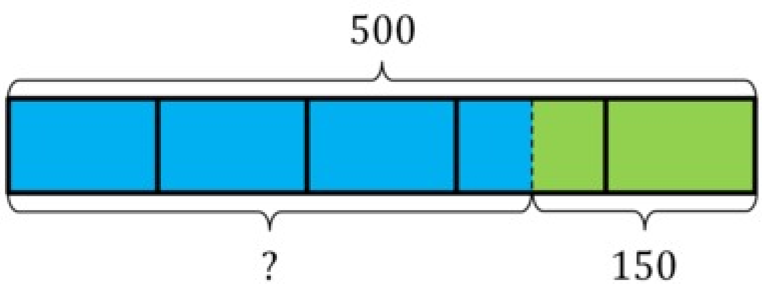
4.MD.A.2

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Solution Possibilities:

Solution: 1 Using cents as a unit

The apples cost 50 cents each so three apples cost 3 × 50 cents which is 150 cents. Margie paid 5 dollars. Since one dollar is 100 cents this means that 5 dollars is 5 × 100 = 500 cents.



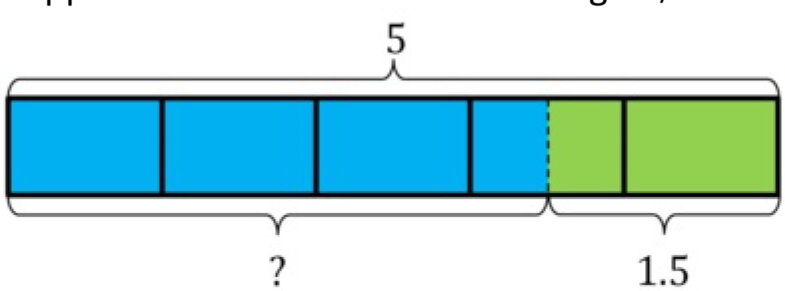
So Margie will get back

500−150=350cents.

This is the same as $3.50.

Solution: 2 Using dollars and fractions

The apples cost 50 cents each. This is 1/2 of a dollar. So two apples will cost 1 dollar and 3 apples cost an extra ½ dollar making 1 1/2 dollars. Margie paid 5 dollars.



So Margie will get back

$$5-1\ ½\ \text{dollars.}$$

Taking away one dollar leaves Margie with 4 dollars and taking away an additional half dollar means Margie will get back 312 dollars or $3.50.

Solution: 3 Using dollars and decimals from the start

The apples cost 50 cents each. Since one cent is 0.01 dollars, 50 cents is 0.50 dollars.

Two apples will cost 1 dollar and 3 apples cost an extra 50 cents making 1.50 dollars.

Margie paid 5 dollars. So Margie will get back

$$5-1.50\text{dollars.}$$

Taking away one dollar leaves Margie with 4 dollars and taking away an additional 0.50 dollars means Margie will get back 3.50 dollars.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Accurately and **quickly** adds and subtracts numbers in different forms. Response includes an **efficient** and logical progression of steps.	Accurately and **in a timely manner** adds and subtracts numbers in different forms. Response includes a **logical** progression of steps	**Accurately** adds and subtracts numbers in different forms. Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	**Accurately** adds and subtracts numbers in different forms with some level of accuracy. Response includes an **incomplete or Illogical** progression of steps.	Does not address task, unresponsive, unrelated or inappropriate.

Core Instructional and Supplemental Materials (K-5)

EUREKA MATH V. 2019

(GREAT MINDS)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K (v. 2019.)	• **Teacher Edition: Module 1-6** • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit	• Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6
1	• **Teacher Edition: Module 1-6** • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit	• Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6
2	• **Teacher Edition: Module 1-8** • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit	• Learn Workbook Set: Module 1-8 • Succeed Workbook Set: Module 1-8 • Practice Workbook, Fluency: Module 1-8
3		
4	• **Teacher Edition: Module 1-7** • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit	• Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
5	• **Teacher Edition: Module 1-7** • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit	• Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
	• **Teacher Edition: Module 1-6** • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit	• Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6

MATH IN FOCUS v. 2015
(HOUGHTON MIFFLIN HARCOURT)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K	• **Teacher Edition (A & B)** • Implementation Guide • Assessment Package • Enrichment Bundle • Extra Practice Set • Teacher and Student Activity Cards • Home -to- School Connection Book • Online Teacher Technology Kit • Big Book Set • Online Interactive Whiteboard Lessons	• Student Edition A – Pt. 1 • Student Edition A – Pt. 2 • Student Edition B – Pt. 1 • Student Edition B – Pt. 2 • Online Student Technology Kit
1	• **Teacher Edition (A & B)** • Implementation Guide • Assessment Package • Enrichment Bundle • Extra Practice Guide • Reteaching Guide • Home -to- School Connection Book • Online Teacher Technology Kit • Fact Fluency • Online Interactive Whiteboard Lessons	• Student Texts (A & B) • Student Workbooks • Online Student Technology Kit • Student Interactivities
2-5	• **Teacher Edition (A & B)** • Implementation Guide • Assessment Package • Enrichment Bundle • Extra Practice Guide • Transition Guides • Reteaching Guide • Home -to- School Connection Book • Online Teacher Technology Kit • Fact Fluency • Online Interactive Whiteboard Lessons	• Student Texts (A & B) • Student Workbooks • Online Student Technology Kit • Student Interactivities

Supplemental Resources

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.